Superstring in a pp-wave background at finite temperature
- TFD approach -

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Abstract

A thermodynamical analysis for the type IIB superstring in a pp-wave background is considered. The thermal Fock space is built and the temperature SUSY breaking appears naturally by analyzing the thermal vacuum. All the thermodynamical quantities are derived by evaluating matrix elements of operators in the thermal Fock space. This approach seems to be suitable to study thermal effects in the BMN correspondence context.

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I. INTRODUCTION

The ADS/CFT correspondence is a concrete realization of a duality relating gravity and Yang-Mills theory. In its strong version this duality asserts that the $\mathcal{N}=4$ $SU(N)$ Super Yang Mills theory and type IIB superstring on $ADS_5 \times S^5$ with N units of five form flux are exactly equivalent [1]. Although a large amount of evidence has emerged, proving this statement turns out to be absolutely hard to come by due to the nonlinearities of the world sheet action.

This scenario has changed with the discovery of a new maximally supersymmetric solution of type IIB supergravity, the pp-waves [2]. Such a background is obtained as the Penrose limit of $ADS_5 \times S^5$ [3]. On the gauge theory side the limit focuses on a set of operators which have R charges J and the conformal dimension Δ satisfying $J \approx \sqrt{N}$ and $\Delta \approx J$, for fixed Yang-Mills coupling and N going to infinity. This set up came to be known as the BMN (Berenstein, Maldacena, Nastase) limit [4]. In addiction it turns out that the superstring is exactly solvable in the pp-wave background and the above mentioned duality is perturbatively accessible from both sides of the correspondence, contrary to the original $ADS_5 \times S^5$ case.

In the case of superstring at finite temperature the BMN correspondence is suitable, since a theory which has a good thermodynamical behaviour is related to another which has not (string theory). Besides to the usual difficulties in studying the thermodynamics of theories which contain gravity, in string theory the level density of states grows exponentially, originating the Hagedorn temperature; at this temperature the free energy diverges [5].

Lately there have been some interesting works studying finite temperature effects of type IIB superstring in the pp-waves background [6], [7], [8], [9], [10]. In a general way these works compute the superstring partition function and the free energy on a torus. The Hagedorn temperature is calculated using the modular properties of the partition function. It was shown that the Hagedorn temperature can be related to a phase transition [6], it may be the deconfinement/confinement phase transition on the gauge side [8]. However, to have a complete understanding of the superstring thermal effects in terms of gauge thermal effects, it is necessary to understand the thermal version of the BMN correspondence and how the temperature SUSY breaking can affect it. Although it has been conjectured that the correspondence exists at finite temperature [11], there are few efforts to test or prove it.

In this letter we present a finite temperature formalism that can be useful for this purpose.

The BMN correspondence is an equivalence of operators and Hilbert spaces of two different theories. The central relation is

$$\frac{H}{\mu} \to \Delta - J, \tag{1}$$

where μ is the only term that comes from the Ramond-Ramond five form and survives to the BMN limit. The string hamiltonian operator H acts on the Fock space built with the string oscillators and gives the energy of each state; $\Delta - J$ acts on the set of gauge invariant operators which survives to BMN limit, giving their conformal dimension minus the J charge. Thus, from the operator correspondence (1), we have a map between the two spaces where the operators act upon as well as a correspondence between the matrix elements in the basis related by this map. How can the temperature be introduced in this scenario?

Due to the operator character of the BMN correspondence, it is useful to introduce the temperature in such a way, that thermodynamics quantities can be derived using operators, Hilbert spaces and matrix elements. This is the case of Thermo Field Dynamics (TFD) developed in [12]. TFD is a real time formalism where the main idea is to interpret the statistical average of any quantity Q over a statistical ensemble as the expectation value of Q in a thermal vacuum

$$Z^{-1}(\beta)\mathrm{T}r[Qe^{-\beta H}] \;=\; \langle 0(\beta)\,|Q|\,0(\beta)\rangle \;, \tag{2}$$

where $\beta = (k_B T)^{-1}$ and k_B is the Boltzmann's constant. All the thermodynamics quantities can be defined as matrix elements of an operator in the thermal vacuum. Also, we have thermal operators acting upon the thermal Fock space with the same properties of the $T = 0$ ones. This comes from the fact that the $T \neq 0$ operators and Fock space are construct from the original $T = 0$ ones by a Bogoliubov transformation. In addiction, the temperature SUSY breaking and the respective Goldstinos come naturally in this formalism [13, 14].

Concerning bosonic string theory TFD was employed to study questions such as string field theory and renormalizability in [15], [16], [17], [18]. The thermal heterotic strings were presented in [19], [20]. Recently, in a set of constructive works [21], [22], [23], [24], [25], [26], TFD has been used to search a microscopic description for the bosonic D-branes thermodynamics.

The aim of this letter is to apply the TFD approach to construct a thermal superstring in the pp-waves background emphasising that the method can be useful to understand thermal effects in the BMN correspondence. In order to take into account the level match condition of the type IIB superstring, it is necessary to reformulate the expectation value of (2) as follows:

$$Z^{-1}(\beta)\int_0^1 d\lambda \mathrm{Tr}[Qe^{-\beta H+2\pi i\lambda P}]=\int_0^1 d\lambda\,\langle 0(\beta,\lambda)\,|Q|\,0(\beta,\lambda)\rangle\,, \tag{3}$$

where P is the momentum operator of the world sheet and the dependence of the thermal vacuum on the lagrange multiplier comes from the Bogoliubov transformation parameter. The action of type IIB superstring in a pp-wave background is only known in the light-cone gauge. In this gauge the energy is $P^0 = P^+ + P^-$, where $P^+\,|\Phi\rangle = p^+\,|\Phi\rangle$, $P^-\,|\Phi\rangle = 1/p^+\,(H_{lc})\,|\Phi\rangle$ and H_{lc} is the light-cone hamiltonian. As pointed out in [27], the one string full partition function, $Z\,(\beta) = Tre^{-\beta P^0}$, is obtained from the transverse ones $z_{lc}\,(\beta/p^+) = Tre^{-\left(\beta/p^+\right)H_{lc}}$, by means of a Laplace transform:

$$Z\,(\beta)=\frac{L}{\sqrt{2\pi}}\int dp^+e^{\beta p^+}z_{lc}\left(\beta/p^+\right), \tag{4}$$

where L is the length of the longitudinal direction. In general, in the imaginary time formalism, the transverse partition function is calculed by evaluating the path integral on a torus. In this letter we will concentrated just in the transverse sector keeping p^+ fixed and look for a thermal vacuum that reproduces the trace in this sector.

This work is organized as follows. The string in the pp-wave background is described in the next section. In Section 3 the TFD approach is carried out to construct the thermal Fock space and the thermal operators for this superstring. Particularly the thermal SUSY breaking and the realization of the Goldstone theorem are showed through the analysis of the thermal vacuum. Finally, in Section 4, the free energy, thermal energy and entropy of the superstring are calculated by evaluating matrix elements of operators in the thermal Fock space; in that section the λ dependence of the Bogoliubov parameter appears naturally when the free energy is minimized. In the last section conclusions and possible extensions of this work are discussed.

II. TYPE IIB SUPERSTRING ON THE PP WAVE BACKGROUND

In this section we summarize some well known results for the type IIB superstring on the pp-waves background. We use the light-cone coordinates $X^{\pm} = \frac{1}{\sqrt{2}}(X^9 \pm X^0)$ and we write the remaining 8 components of the spinors (after Kappa symmetry fixing) as S^a, $\bar{S}^b$, composing the $\mathbf{8_s}$ representation of SO(8). The chiral representation of SO(8) gamma matrices is used.

The metric of the pp-wave is

$$ds^2 = 2dx^+dx^- - \mu^2 x^I x^I dx^I dx^I + dx^I dx^I, \qquad I = i, i', \tag{5}$$

$i = 1, ...4$, $i' = 5, ...8$. It is obtained from $ADS_5 \times S^5$ by a Penrose limit, where the only surviving components of the Ramond-Ramond five form are: $F_{+1234} = F_{+5678} = \mu$. This metric preserves all the 32 supersymmetries of the type IIB superstring but breaks the $SO(8)$ down to $SO(4) \times SO(4)$. The light-cone gauge (Kappa) fixed action for type IIB Green-Schwarz superstring on this geometry is [28]

$$S = \frac{1}{2\pi\alpha'} \int d\sigma^2 \left(\frac{1}{2}\partial_+ X^I \partial_- X^I - \frac{1}{2} m^2 (X^I)^2 + iS^a \partial_+ S^a + i\bar{S}^a \partial_- \bar{S}^a - 2imS^a \Pi_{ab} \bar{S}^b \right), \tag{6}$$

where $\partial_\pm = \partial_\tau \pm \partial_\sigma$ and Π is a traceless tensor defined as $\Pi = \gamma^1\gamma^2\gamma^3\gamma^4$. The mass parameter m is defined as $m = \mu\,\alpha' p^+$. The solutions of the equations of motion with periodic boundary conditions are [29]

$$\begin{aligned} X^I = x_0^I \cos(m\tau) + \frac{\alpha'}{m} p_0^I \sin(m\tau) \; + \sqrt{\frac{\alpha'}{2}} \sum_{n>0} \frac{1}{\sqrt{\o_n}} \Big[&\left(a_n^I e^{-i(\omega_n \tau - k_n \sigma)} + a_n^{\dagger\, I} e^{i(\omega_n \tau - k_n \sigma)} \right) \\ &+ \left(\bar{a}_n^I e^{-i(\omega_n \tau + k_n \sigma)} + \bar{a}_n^{\dagger\, I} e^{i(\omega_n \tau + k_n \sigma)} \right) \Big] , \end{aligned} \tag{7}$$

and

$$\begin{aligned} S^a = \cos(m\tau) S_0^a + \sin(m\tau) \Pi_{ab} \bar{S}_0^b + \sum_{n>0} c_n \Big[& S_n^a e^{-i(\omega_n \tau - k_n \sigma)} \; + S_n^{\dagger a} e^{i(\o_n \tau - k_n \sigma)} \\ &+ i \frac{\omega_n - k_n}{m} \Pi_{ab} \left(\bar{S}_n^b e^{-i(\omega_n \tau + k_n \sigma)} - \bar{S}_n^{\dagger b} e^{i(\o_n \tau + k_n \sigma)} \right) \Big] , \end{aligned} \tag{8}$$

$$\begin{aligned} \bar{S}^a = \cos(m\tau) \bar{S}_0^a - \sin(m\tau) \Pi_{ab} S_0^b + \sum_{n>0} c_n \Big[& \bar{S}_n^a e^{-i(\omega_n \tau + k_n \sigma)} \; + \bar{S}_n^{\dagger a} e^{i(\o_n \tau + k_n \sigma)} \\ &- i \frac{\omega_n - k_n}{m} \Pi_{ab} \left(S_n^b e^{-i(\omega_n \tau - k_n \sigma)} - S_n^{\dagger b} e^{i(\o_n \tau - k_n \sigma)} \right) \Big] , \end{aligned} \tag{9}$$

where we set:

$$\o_n = \sqrt{m^2 + k_n^2}, \qquad c_n = \frac{1}{\sqrt{1 + (\frac{\o_n - k_n}{m})^2}}, \qquad k_n = 2\pi n. \tag{10}$$

The canonical quantization gives the standard commutator and anti-commutator relations of harmonic oscillator for a_n, $a_n^\dagger$ and S_n, $S_n^\dagger$, respectively, and the same for "bar" operators. The zero mode part is written as follows

$$\begin{aligned} a_0^I &= \frac{1}{\sqrt{2m}}(p_0^I - imx_0^I), \qquad {a_0^\dagger}^I = \frac{1}{\sqrt{2m}}(p_0^I + imx_0^I), \\ S_\pm^a &= \frac{1}{2}\left(1 \pm \Pi\right)_{ab} \frac{1}{\sqrt{2m}}\left(S_0^b \pm i\bar{S}_0^b\right), \qquad {S_\pm^\dagger}^a = \frac{1}{2}\left(1 \pm \Pi\right)_{ab} \frac{1}{\sqrt{2m}}\left(S_0^b \mp i\bar{S}_0^b\right), \end{aligned} \tag{11}$$

which satisfies

$$\begin{aligned} \left[a_0^I, {a_0^\dagger}^J\right] &= \delta^{IJ}, \qquad \left[{a_0^\dagger}^I, {a_0^\dagger}^J\right] = \left[a_0^I, a_0^J\right] = 0, \\ \left\{S_\pm^a, {S_\pm^\dagger}^b\right\} &= \delta^{ab}, \qquad \left\{S_\pm^a, S_\pm^b\right\} = \left\{{S_\pm^\dagger}^a, {S_\pm^\dagger}^b\right\} = 0. \end{aligned} \tag{12}$$

The light-cone hamiltonian is calculated in a standard way and it is written as

$$p_+ H = m\left({a_0^\dagger}^I a_0^I + {S_+^\dagger}^a S_+^a + {S_-^\dagger}^a S_-^a\right) + \sum_{n>0} \omega_n \left({a_n^\dagger}^I a_n^I + {\bar{a}_n^\dagger}^I \bar{a}_n + {S_n^\dagger}^a S_n^a + {\bar{S}_n^\dagger}^a \bar{S}_n^a\right). \tag{13}$$

In addiction to the time translations generated by the hamiltonian, the action has 29 more bosonic symmetries (generated by P^+, P^I and by the rotations J^{+I}, J^{ij}, $J^{i',j'}$) and 32 supersymmetries. The fermionic set of generators has 16 kinematical supercharges, that belong to $\mathbf{8_s}$ of $SO(8)$ and changes the polarizations of the fields. The remaining 16 fermionic symmetries are the dynamical supercharges, that transform the fields of the same supermultiplet and belong to $\mathbf{8_c}$ of $SO(8)$. While the kinematical supercharge does not commute with the hamiltonian, the dynamical one does, providing a supersymmetric spectrum. The dynamical supercharges can be written as $Q_{\dot{\alpha}}^\pm = Q_{\dot{\alpha}} \pm \bar{Q}_{\dot{\alpha}}$, where

$$\begin{aligned} \frac{\sqrt{p^+}}{2^{1/4}} Q_{\dot{a}} &= P_0^I \left(\gamma^I S_0\right)_{\dot{a}} - mX_0^I \left(\gamma \Pi \bar{S}_0\right)_{\dot{a}} \\ &+ \sum_{n>0} \left[(\sqrt{2\omega_n} c_n \left({a_n^\dagger}^I \gamma^I S_n + a_n^I \gamma^I S_n^\dagger\right)_{\dot{a}} + \frac{im}{\sqrt{2\omega_n} c_n} \left(\gamma \Pi\right)_{\dot{a}b} \left({\bar{a}_n^\dagger}^I \bar{S}_n^b - \bar{a}_n^I {\bar{S}_n^\dagger}^b\right) \right], \end{aligned} \tag{14}$$

and $\bar{Q}_{\dot{a}}$ can be obtained from $Q_{\dot{a}}$ replacing "bar" variables by non-bar variables and i by $-i$, while the kinematic supercharges are $Q \approx S_0$ and $\bar{Q} \approx \bar{S}_0$.

Finally, we can choose the vacuum $|0, p^+\rangle$ as defined by

$$
\begin{aligned}
S_n \left|0, p^+\right\rangle &= \bar{S}_n \left|0, p^+\right\rangle = 0 \qquad n > 0, \\
a_n^I \left|0, p^+\right\rangle &= \bar{a}_n^I \left|0, p^+\right\rangle = 0 \qquad n > 0, \\
S_\pm \left|0, p^+\right\rangle &= a_0^I \left|0, p^+\right\rangle = 0.
\end{aligned} \tag{15}
$$

The hamiltonian and the dynamical supercharges annihilate the vacuum as a signal of supersymmetry. Following the BMN dictionary, the vacuum has zero energy and is related to an operator in the gauge side with zero value for $\Delta - J$:

$$
\left|0, p^+\right\rangle \to O^J(0) \left|vac\right\rangle, \qquad O(x) = \frac{1}{\sqrt{JN^J}} TrZ^J, \tag{16}
$$

where $|vac\rangle$ is the Yang-Mills vacuum and O^J is composed of two out of the six scalar fields of the $N = 4$ super Yang Mills multiplet: $Z = \frac{1}{2}(\phi^5 + i\phi^6)$. The trace is taken over the $SU(N)$ index. The next section is devoted to construct a thermal vacuum for the string, that can be useful to understand how the above dictionary is affected by the temperature.

III. TFD APPROACH

Let us now apply the TFD approach to construct the thermal Fock space for superstring on a pp-wave background. Following Umezawa, to provide enough room to accommodate the thermal properties of the system, we have first to duplicate the degrees of freedom. To this end we construct a copy of the original Hilbert space, denoted by $\widetilde{H}$. The Tilde Hilbert space is built with a set of oscillators: $\tilde{a}_0$, $\tilde{S}_\pm$, $\tilde{a}_n$, $\tilde{\bar{a}}_n$, $\tilde{S}_n$, $\tilde{\bar{S}}_n$ that have the same (anti-) commutation properties as the original ones. The operators of the two systems commute among themselves and the total Hilbert space is the tensor product of the two spaces.

We can now construct the thermal vacuum. This is achieved by implementing a Bogoliubov transformation in the total Hilbert space. The transformation generator is given by

$$
G = G^B + G^F, \tag{17}
$$

for

$$
\begin{aligned}
G^B &= G_0^B + \sum_{n=1} \left(G_n^B + \bar{G}_n^B\right), && (18) \\
G^F &= G_+^F + G_-^F + \sum_{n=1} \left(G_n^F + \bar{G}_n^F\right), && (19)
\end{aligned}
$$

where

$$
\begin{aligned}
G_0^B &= -i\theta_0^B \left(a_0 \cdot \tilde{a}_0 - \tilde{a}_0^\dagger \cdot a_0^\dagger\right), &(20)\\
G_n^B &= -i\theta_n^B \left(a_n \cdot \tilde{a}_n - \tilde{a}_n^\dagger \cdot a_n^\dagger\right), &(21)\\
\bar{G}_n^B &= -i\bar{\theta}_n^B \left(\bar{a}_n \cdot \tilde{\bar{a}}_n - \tilde{\bar{a}}_n^\dagger \cdot \bar{a}_n^\dagger\right), &(22)\\
G_\pm^F &= -i\theta_\pm^F \left(\widetilde{S}_\pm \cdot S_\pm - S_\pm^\dagger \cdot \widetilde{S}_\pm^\dagger\right), &(23)\\
G_n^F &= -i\theta_n^F \left(\widetilde{S}_n \cdot S_n - S_n^\dagger \cdot \widetilde{S}_n^\dagger\right), &(24)\\
G_n^F &= -i\bar{\theta}_n^F \left(\widetilde{\bar{S}}_n \cdot \bar{S}_n - \bar{S}_n^\dagger \cdot \widetilde{\bar{S}}_n^\dagger\right). &(25)
\end{aligned}
$$

Here, the labels B and F specifies fermions and bosons, the dots represent the inner products and θ, $\bar{\theta}$ are real parameters. In the thermal equilibrium they are related to the Bose-Einstein and Fermi-Dirac distribution of the oscillator n as we will see. The thermal vacuum is given by the following relation

$$
\begin{aligned}
|0\,(\theta)\rangle &= e^{-iG}\,|0\rangle\!\rangle \\
&= \left(\frac{1}{\cosh(\theta_0^B)}\right)^8 \left(\cos(\theta_+^F)\right)^4 \left(\cos(\theta_-^F)\right)^4 e^{\tanh\left(\theta_0^B\right)\left(a_0^\dagger\cdot\tilde{a}_0^\dagger\right)} e^{\tan\left(\theta_+^F\right)\left(S_+^\dagger\cdot\tilde{S}_+^\dagger\right)+\tan\left(\theta_+^F\right)\left(S_-^\dagger\cdot\tilde{S}_-^\dagger\right)} \\
&\times \prod_{n=1}\left[\left(\frac{1}{\cosh(\theta_n^B)}\right)^8 \left(\frac{1}{\cosh(\bar{\theta}_n^B)}\right)^8 e^{\tanh\left(\theta_n^B\right)\left(a_n^\dagger\cdot\tilde{a}_n^\dagger\right)+\tanh\left(\bar{\theta}_n^B\right)\left(\bar{a}_n^\dagger\cdot\tilde{\bar{a}}_n^\dagger\right)}\right. \\
&\times \left.\left(\cos(\theta_n^F)\right)^8 \left(\cos(\bar{\theta}_n^F)\right)^8 e^{\tan\left(\theta_n^F\right)\left(S_n^\dagger\cdot\tilde{S}_n^\dagger\right)+\tan\left(\bar{\theta}_n^F\right)\left(\bar{S}_n^\dagger\cdot\tilde{\bar{S}}_n^\dagger\right)}\right] |0\rangle\!\rangle\,. \qquad (26)
\end{aligned}
$$

The creation and annihilation operators at $T \neq 0$ are given by the Bogoliubov transformation as follows

$$
\begin{aligned}
S_\pm^a\left(\theta_\pm^F\right) &= e^{-iG} S_\pm^a e^{iG} = \cos\left(\theta_\pm^F\right) S_\pm^a - \sin\left(\theta_\pm^F\right) \widetilde{S}_\pm^{\dagger\,a}, &(27)\\
S_n^a\left(\theta_n^F\right) &= e^{-iG} S_n^a e^{iG} = \cos\left(\theta_n^F\right) S_n^a - \sin\left(\theta_n^F\right) \widetilde{S}_n^{\dagger\,a}, &(28)\\
\bar{S}_n^a\left(\bar{\theta}_n^F\right) &= e^{-iG} \bar{S}_n^a e^{iG} = \cos\left(\bar{\theta}_n^F\right) \bar{S}_n^a - \sin\left(\bar{\theta}_n^F\right) \widetilde{\bar{S}}_n^{\dagger\,a}, &(29)\\
a_0^I\left(\theta_0^B\right) &= e^{-iG} a_0^I e^{iG} = \cosh\left(\theta_0^B\right) a_0^I - \sinh\left(\theta_0^B\right) \widetilde{a}_0^{\dagger\,I}, &(30)\\
a_n^I\left(\theta_n^B\right) &= e^{-iG} a_n^I e^{iG} = \cosh\left(\theta_n^B\right) a_n^I - \sinh\left(\theta_n^B\right) \widetilde{a}_n^{\dagger\,I}, &(31)\\
\bar{a}_n^I\left(\bar{\theta}_n^B\right) &= e^{-iG} \bar{a}_n^I e^{iG} = \cosh\left(\bar{\theta}_n^B\right) \bar{a}_n^I - \sinh\left(\theta_n^B\right) \widetilde{\bar{a}}_n^{\dagger\,I}. &(32)
\end{aligned}
$$

These operators annihilate the state written in (26) defining it as the vacuum. The creation operators are obtained from the above list by hermitian conjugation. The tilde counterparts

can be obtained using the tilde conjugation rules defined in [12]. As the transformation generator defined in $(17)-(25)$ is hermitian and changes the signal under tilde conjugation, one can check that the vacuum is invariant under this conjugation.

The thermal Fock space is constructed from the vacuum (26) by applying the thermal creation operators. As the Bogoliubov transformation is canonical, the thermal operators obey the same (anti-) commutators relations as the operators at $T = 0$.

The hamiltonian plays an important rôle in the BMN correspondence, so a natural question is what is the rôle it plays in the thermal Fock space. It is easy to see that thermal states are not eigenstates of the original hamiltonian but they are eigenstates of the combination:

$$\widehat{H} = H - \widetilde{H}, \tag{33}$$

in such a way that $\widehat{H}$ plays the rôle of the hamiltonian generating the temporal translation in the thermal Fock space. Using the commutation relations we can prove that the Heisenberg equations are satisfied replacing H and $\widetilde{H}$ by $\widehat{H}$. Also we have $\widehat{Q}_{\dot{a}}$ and $\widehat{\bar{Q}}_{\dot{a}}$ defined in a similar way as $\widehat{H}$ in (33). These new supercharges realize the same supersymmetry algebra as the supercharges at $T = 0$. However, this fact does not imply that the supersymmetry remains at finite temperature. In fact, in the TFD approach, thermal effects are observed when one considers $T = 0$ operators expectation values on the thermal Fock space. For $T = 0$ the dynamical supercharge commutes with the hamiltonian and as a consequence of the supersymmetry algebra annihilates the vacuum. Now, by applying the $Q^{+}_{\dot{a}}$ operator on the thermal vacuum, we get:

$$\begin{aligned}\frac{Q^{+}_{\dot{a}}}{2^{1/4}\sqrt{\mu}}\left|0\left(\theta\right)\right\rangle \;=\; & \Big[a^{\dagger\,I}_{0}\left(\theta^{B}_{0}\right)\left(\gamma^{I}\tilde{S}^{\dagger}_{+}\left(\theta^{F}_{+}\right)\right)_{\dot{a}}\cosh\left(\theta^{B}_{0}\right)\sin\left(\theta^{F}_{+}\right) \\ & +\tilde{a}^{I}_{0}\left(\theta\right)\left(\gamma^{I}S^{\dagger}_{-}\left(\theta^{F}_{-}\right)\right)_{\dot{a}}\sinh\left(\theta^{B}_{0}\right)\cos\left(\theta^{F}_{-}\right)+\text{osc.terms}\Big]\left|0\left(\theta\right)\right\rangle .\end{aligned} \tag{34}$$

These new excitations generated by $Q_{\dot{a}}$ have interesting properties. If we apply the hamiltonian $\widehat{H}$ in this state we have:

$$\widehat{H}\left(Q_{\dot{a}}\left|0\left(\theta\right)\right\rangle\right) = 0, \tag{35}$$

so these excitations are in fact massless excitations with respect to the hamiltonian $\widehat{H}$. For each supersymmetric oscillator n we have a massless excitation proportional to $a^{\dagger\,I}_{n}\left(\theta^{B}_{n}\right)\left(\gamma^{I}\tilde{S}_{n}\left(\theta^{F}_{n}\right)\right) + \tilde{a}_{n}\left(\theta^{B}_{n}\right)^{\dagger\,I}\left(\gamma^{I}S^{\dagger}_{n}\left(\theta^{F}_{n}\right)\right)$ plus "bar" variables. This combination is called super pair and realizes the Goldstone theorem for the supersymmetry breaking gen-

erated by the temperature [13, 14]. They play the rôle of the Goldstinos, although they are not really particles since there are no interactions.

In this section we have constructed both the thermal vacuum and Fock space and demonstrated the breaking of $T = 0$ SUSY. Next section is devoted to find thermodynamic quantities and analyze the thermodynamics of the superstring on a pp-wave background.

IV. THERMODYNAMICAL ANALYSIS

In this section the TFD approach will be used to compute thermodynamical quantities by evaluating matrix elements of operators in the thermal Fock space. It was appointed out by Polchinski [30] that, in the one-string sector, the torus path integral computation of the free energy coincides with what we would obtain by adding the contributions from different states of the spectrum to the free energy. Here the free energy is obtained from the knowledge of the thermal energy and entropy operators.

The energy operator is such that the level matching condition must be implemented. The way we proceed is to consider a shifted hamiltonian in the sense of Ref. [31], as follows

$$H = \frac{1}{p+}\left[mN_0 + \sum_{n=1}\omega_n\left(N_n + \bar{N}_n\right)\right] + \frac{1}{\beta}i\lambda\sum_{n=1}k_n\left(N_n - \bar{N}_n\right), \tag{36}$$

where

$$N_0 = N_0^B + N_+^F + N_-^F, \tag{37}$$

and

$$N_n = N_n^B + N_n^F, \qquad \bar{N}_n = \bar{N}_n^B + \bar{N}_n^F. \tag{38}$$

Computing the expectation value of (36) in the thermal vacuum (26), the following result arises

$$\begin{aligned}
E &\equiv \int_0^1 d\lambda \left\langle 0\left(\theta_\lambda\right)\right| H \left|0\left(\theta_\lambda\right)\right\rangle \\
&= \int_0^1 d\lambda\left[\frac{m}{p+}\left[8\sinh^2\left(\theta_0^B\right) + 4\sin^2\left(\theta_+^F\right) + 4\sin^2\left(\theta_-^F\right)\right]\right. \\
&+\frac{8}{p+}\sum_{n=1}\omega_n\left[\sinh^2\left(\theta_n^B\right) + \sin^2\left(\theta_n^F\right) + \sinh^2\left(\overline{\theta}_n^B\right) + \sin^2\left(\overline{\theta}_n^F\right)\right] \\
&\left.+\frac{8\lambda i}{\beta}\left[\sum_{n=1}k_n\left(\sinh^2\left(\theta_n^B\right) + \sin^2\left(\theta_n^F\right) - \sinh^2\left(\overline{\theta}_n^B\right) - \sin^2\left(\overline{\theta}_n^F\right)\right)\right]\right],
\end{aligned} \tag{39}$$

where θ_λ just specifies the lagrange multiplier dependence of the Bogoliubov parameter. Concerning the entropy operator, an extension to that presented in [22] is necessary in order to include the fermionic degrees of freedom. Namely,

$$K = K^B + K^F, \tag{40}$$

where the boson contribution is given by

$$\begin{aligned} K^B &= -\left\{a_0^\dagger \cdot a_0 \ln\left(\sinh^2\left(\theta_0^B\right)\right) - a_0 \cdot a_0^\dagger \ln\left(\cosh^2\left(\theta_0^B\right)\right)\right\} \\ &\quad - \sum_{n=1} \left\{a_n^\dagger \cdot a_n \ln\left(\sinh^2\left(\theta_n^B\right)\right) - a_n \cdot a_n^\dagger \ln\left(\cosh^2\left(\theta_n^B\right)\right)\right\} \\ &\quad - \sum_{n=1} \left\{\overline{a}_n^\dagger \cdot \overline{a}_n \ln\left(\sinh^2\left(\overline{\theta}_n^B\right)\right) - \overline{a}_n \cdot \overline{a}_n^\dagger \ln\left(\cosh^2\left(\overline{\theta}_n^B\right)\right)\right\}, \end{aligned} \tag{41}$$

and

$$\begin{aligned} K^F &= -\left\{S_+^\dagger \cdot S_+ \ln\left(\sin^2\left(\theta_+^F\right)\right) + S_+ \cdot S_+^\dagger \ln\left(\cos^2\left(\theta_+^F\right)\right)\right\} \\ &\quad - \left\{S_-^\dagger \cdot S_- \ln\left(\sin^2\left(\theta_-^F\right)\right) + S_- \cdot S_-^\dagger \ln\left(\cos^2\left(\theta_-^F\right)\right)\right\} \\ &\quad - \sum_{n=1} \left\{S_n^\dagger \cdot S_n \ln\left(\sin^2\left(\theta_n^F\right)\right) + S_n \cdot S_n^\dagger \ln\left(\cos^2\left(\theta_n^F\right)\right)\right\} \\ &\quad - \sum_{n=1} \left\{\overline{S}_n^\dagger \cdot \overline{S}_n \ln\left(\sin^2\left(\overline{\theta}_n^F\right)\right) + \overline{S}_n \cdot \overline{S}_n^\dagger \ln\left(\cos^2\left(\overline{\theta}_n^F\right)\right)\right\}, \end{aligned} \tag{42}$$

is the entropy operator for the fermionic sector. The evaluation of the entropy operator on the thermal vacuum leads to the following result

$$\begin{aligned} S &\equiv \int_0^1 d\lambda \left\langle 0\left(\theta_\lambda\right)\right| K \left|0\left(\theta_\lambda\right)\right\rangle \\ &= \int_0^1 d\lambda \left\{-8\left[\sinh^2\left(\theta_0^B\right)\ln\left(\tanh^2\left(\theta_0^B\right)\right) - \ln\left(\cosh^2\left(\theta_0^B\right)\right)\right]\right. \\ &\quad -4\left[\sin^2\left(\theta_+^F\right)\ln\left(\tan^2\left(\theta_+^F\right)\right) + \ln\left(\cos^2\left(\theta_+^F\right)\right)\right] \\ &\quad -4\left[\sin^2\left(\theta_-^F\right)\ln\left(\tan^2\left(\theta_-^F\right)\right) + \ln\left(\cos^2\left(\theta_-^F\right)\right)\right] \\ &\quad -8\sum_{n=1}\left[\sinh^2\left(\theta_n^B\right)\ln\left(\tanh^2\left(\theta_n^B\right)\right) - \ln\left(\cosh^2\left(\theta_n^B\right)\right)\right] \\ &\quad -8\sum_{n=1}\left[\sinh^2\left(\overline{\theta}_n^B\right)\ln\left(\tanh^2\left(\overline{\theta}_n^B\right)\right) - \ln\left(\cosh^2\left(\overline{\theta}_n^B\right)\right)\right] \\ &\quad -8\sum_{n=1}\left[\sin^2\left(\theta_n^F\right)\ln\left(\tan^2\left(\theta_n^F\right)\right) + \ln\left(\cos^2\left(\theta_n^F\right)\right)\right] \\ &\quad \left. -8\sum_{n=1}\left[\sin^2\left(\overline{\theta}_n^F\right)\ln\left(\tan^2\left(\overline{\theta}_n^F\right)\right) + \ln\left(\cos^2\left(\overline{\theta}_n^F\right)\right)\right]\right\}. \end{aligned} \tag{43}$$

Now one can construct the potential

$$F=E-\frac{1}{\beta}S, \tag{44}$$

where explicitly we have the following expression

$$\begin{aligned}
F \;=\; & \int_0^1 d\lambda \left\{ \frac{m}{p+}\left[8\sinh^2\left(\theta_0^B\right)+4\sin^2\left(\theta_+^F\right)+4\sin^2\left(\theta_-^F\right)\right] \right.\\
& +\frac{1}{\beta}\left[8\sinh^2\left(\theta_0^B\right)\ln\left(\tanh^2\left(\theta_0^B\right)\right)-8\ln\left(\cosh^2\left(\theta_0^B\right)\right)+4\sin^2\left(\theta_+^F\right)\ln\left(\tan^2\left(\theta_+^F\right)\right)\right.\\
& \left.+4\ln\left(\cos^2\left(\theta_+^F\right)\right)+4\sin^2\left(\theta_-^F\right)\ln\left(\tan^2\left(\theta_-^F\right)\right)+4\ln\left(\cos^2\left(\theta_-^F\right)\right)\right]\\
& +8\sum_{n=1}\left\{\left(\frac{\omega_n}{p+}+\frac{i\lambda k_n}{\beta}\right)\left[\sinh^2\left(\theta_n^B\right)+\sin^2\left(\theta_n^F\right)\right]\right.\\
& +\left(\frac{\omega_n}{p+}-\frac{i\lambda k_n}{\beta}\right)\left[\sinh^2\left(\overline{\theta}_n^B\right)+\sin^2\left(\overline{\theta}_n^F\right)\right]+\frac{1}{\beta}\left[\sinh^2\left(\theta_n^B\right)\ln\left(\tanh^2\left(\theta_n^B\right)\right)\right.\\
& -\ln\left(\cosh^2\left(\theta_n^B\right)\right)+\sinh^2\left(\overline{\theta}_n^B\right)\ln\left(\tanh^2\left(\overline{\theta}_n^B\right)\right)\\
& -\ln\left(\cosh^2\left(\overline{\theta}_n^B\right)\right)+\sin^2\left(\theta_n^F\right)\ln\left(\tan\left(\theta_n^F\right)\right)\\
& \left.\left.\left.+\ln\left(\cos^2\left(\theta_n^F\right)\right)+\sin^2\left(\overline{\theta}_n^F\right)\ln\left(\tan\left(\overline{\theta}_n^F\right)\right)+\ln\left(\cos^2\left(\overline{\theta}_n^F\right)\right)\right]\right\}\right\}.
\end{aligned} \tag{45}$$

Minimizing the potential F with respect to θ we find the explicit dependence of these parameters in relation to ω_n, β and λ. In this way we have

$$\sinh^2\left(\theta_0^B\right)=\frac{1}{e^{\frac{\beta m}{p+}}-1}, \qquad \sin^2\left(\theta_\pm^F\right)=\frac{1}{e^{\frac{\beta m}{p+}}-1}, \tag{46}$$

for the zero modes, and

$$\begin{aligned}
\sinh^2\left(\theta_n^B\right) &= \frac{1}{e^{\frac{\beta\omega_n}{p+}+i\lambda k_n}-1}, \qquad \sinh^2\left(\overline{\theta}_n^B\right)=\frac{1}{e^{\frac{\beta\omega_n}{p+}-i\lambda k_n}-1},\\
\sin^2\left(\theta_n^F\right) &= \frac{1}{e^{\frac{\beta\omega_n}{p+}+i\lambda k_n}+1}, \qquad \sin^2\left(\overline{\theta}_n^F\right)=\frac{1}{e^{\frac{\beta\omega_n}{p+}-i\lambda k_n}+1},
\end{aligned} \tag{47}$$

for the others. These expressions fix the thermal vacuum (26) as those that reproduce the trace over the transverse sector.

Note that substituting the above results in expression (45) we find

$$F=-\frac{1}{\beta}\int_0^1 d\lambda\ln\prod_{n=\mathbb{Z}}\left[\frac{1+e^{-\frac{\beta\omega_n}{p+}+i\lambda k_n}}{1-e^{-\frac{\beta\omega_n}{p+}+i\lambda k_n}}\right]^8. \tag{48}$$

This expression is the TFD answer for the transverse free energy. To make contact with other formalisms, define a potential $f\left(\lambda\right)$ such that

$$F=\int_0^1 d\lambda f\left(\lambda\right), \tag{49}$$

where $f(\lambda)$, given by the equation (48), can be written as

$$f(\lambda) = -\frac{1}{\beta} \ln\left(z_{lc}(\beta/p^{+}, \lambda)\right), \tag{50}$$

and $z_{lc}(\beta/p^{+}, \lambda)$ is the tranverse partition function calculated in [6].

V. CONCLUSIONS

In this letter we show that the Thermo Field Dynamics approach is very useful to calculate thermodynamical quantities for the superstring in a pp-wave background. The main characteristic of this approach is the construction of a thermal Fock space and thermal operators. Owing to the operator characteristics of BMN correspondence, the TFD approach can be a powerful method to set up a possible thermal BMN correspondence.

The transverse thermal energy of the superstring is derived by evaluating the matrix elements of the $T = 0$ hamiltonian in the thermal vacuum. In the same way the entropy and free energy are computed. These results can be obtained in the imaginary time formalism by evaluating the partition function on the torus. Here it is necessary to emphasize that in the TFD approach the world sheet in defined on the sphere, and our results can be compared with those coming from the torus as a consequence of the thermal Bogoliubov transformation. This seems to avoid the problems of the Wick rotation on a pp-wave background pointed out, for example, in [32].

As a consequence of the thermal breaking of supersymmetry, the dynamical supercharge does not annihilate the vacuum anymore. We have shown that the dynamical supercharge excites massless modes in the vacuum, that play the rôle of the Goldstinos [13, 14].

There are many possible extensions of this work. The most direct one is to use the TFD algorithm to construct a thermal Hilbert space on the gauge side of the BMN correspondence. We expect to be able to construct a thermal Fock space and a well defined $\hat{\Delta} - \hat{J}$ operator that can be related with the $\widehat{H}$ presented in (33). On the string side we can use the TFD perturbation theory defined in [12] to go beyond the one-string sector and include the string field cubic interactions [33], [34], [35], [36], which is a very hard task in the usual imaginary time formalism. In another direction, we have shown that by evaluating expected values on a sphere, we reproduce the usual torus results of the imaginary time formalism. These results correspond to the sum of the free energy of each string mode, as was pointed out in [30]. We

can use further the operator method defined in [37] to evaluate the expectation values on a torus and get quantum string corrections to the thermal energy derived in this letter. Finally, as this formalism is a real time formalism, it is possible go out of thermal equilibrium, which can be very useful to understand the thermodynamics of the early universe.

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